Nested Leaky-Box Model

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Primary Nuclei solution

- The equilibrium density N_i of a primary cosmic ray species i in the Galaxy is given by

$$n_i = Q_i \tau_{\mathrm{G}} \tag{1}$$

where τ_{G} is the energy-independent galactic escape time and Q_i is the galactic emissivity. We assume NO inelastic interactions of primaries in the cocoon.

- The emissivity as a function of the kinetic energy per nucleon E is given by

$$Q_i = \rho_s q_i(E) \tag{2}$$

where $\rho_s = \mathcal{R}/V_{\mathrm{G}}$ is the source rate density and q_i is the injection spectrum.

- The injected spectrum can be parametrized as

$$q_i(E) = q_{0,i} \left(\frac{E}{E_0} \right)^{-\alpha_i} \left[1 + \left(\frac{E}{E_{b,i}} \right)^{\frac{\Delta\alpha}{s}} \right]^s \tag{3}$$

and normalized such that

$$\int_{E_0}^{\infty} dE \, A E q_p(E) = \xi E_{\mathrm{CR}} \rightarrow q_{0,i} \simeq \frac{\xi_i E_{\mathrm{CR}} (\alpha_i - 2)}{A E_0^2} \tag{4}$$

- the equilibrium density can be thereby written

$$N_i = \frac{\mathcal{R} \tau_{\mathrm{G}}}{V_{\mathrm{G}}} q_i(E) = \frac{\xi E_{\mathrm{SN}} N(E) \mathcal{R}}{2\pi R_d^2} \frac{\tau_{\mathrm{G}}}{h} \tag{5}$$

Secondary nuclei solution

- Here we distinguish two contributions, first the standard Galactic component

$$Q_j^{\mathrm{G}}(E) = N_i(E) n_{\mathrm{G}} c \sigma_{i\to j} \quad (6)$$

  where N_i is the equilibrium density of a primary species i.

- the second contribution is given by the cocoons, here the source term is given by

$$Q_j^{\mathrm{C}}(E) = \rho_s q_i(E) \tau_{\mathrm{C}}(E) n_{\mathrm{C}} c \sigma = N_i(E) n_{\mathrm{C}} c \sigma_{i\to j} \frac{\tau_{\mathrm{C}}(E)}{\tau_{\mathrm{G}}} \quad (7)$$

  as it depends on the primary equilibrium in the cocoon.

- The secondary equilibrium spectrum

$$N_j = (Q_j^{\mathrm{G}} + Q_j^{\mathrm{C}}) \tau_{\mathrm{G}} \quad (8)$$

- From that follows

$$\frac{N_j}{N_i} = c \sigma_{i\to j} n_{\mathrm{G}} \left[\tau_{\mathrm{G}} + \frac{n_{\mathrm{C}}}{n_{\mathrm{G}}} \tau_{\mathrm{C}}(E) \right] \quad (9)$$

- Or in terms of grammage $\chi_i^{\mathrm{G}} = m_p c n_{\mathrm{G}} \tau_{\mathrm{G}}$

$$\frac{N_j}{N_i} = \frac{\sigma_{i\to j}}{m_p} \left[\chi_{\mathrm{G}} + \chi_{\mathrm{C}}(E) \right] \quad (10)$$

- It can be easily generalized to the case of multiple primaries, in case of two:

$$\frac{N_j}{N_i} = \left(\frac{\sigma_{i\to j}}{m_p} + \frac{N_k}{N_i} \frac{\sigma_{k\to j}}{m_p} \right) \left[\chi_{\mathrm{G}} + \chi_{\mathrm{C}}(E) \right] \quad (11)$$

C/O

- Assume O purely primary, while C has a primary component plus a secondary one.
- Oxygen is

$$N_{\mathrm{O}} = \frac{\mathcal{R}\tau_{\mathrm{G}}}{V_{\mathrm{G}}} q_{\mathrm{O}}(E) \tag{12}$$

- Carbon is

$$N_{\mathrm{C}} = \frac{\mathcal{R}\tau_{\mathrm{G}}}{V_{\mathrm{G}}} q_{\mathrm{C}}(E) + \frac{\sigma_{\mathrm{O}\to\mathrm{C}}}{m_p} \left[\chi_{\mathrm{G}} + \chi_{\mathrm{c}}(E)\right] N_{\mathrm{O}} \tag{13}$$

- The ratio is

$$\frac{N_{\mathrm{C}}}{N_{\mathrm{O}}} = \frac{q_{\mathrm{C}}}{q_{\mathrm{O}}} + \frac{\sigma_{\mathrm{O}\to\mathrm{C}}}{m_p} \left[\chi_{\mathrm{G}} + \chi_{\mathrm{c}}(E)\right] \tag{14}$$

Model parameters

- As in Cowsik et al. the shape of τ_{C} is taken ad mentulam canis as

$$\chi_{\mathrm{C}}(E) = \chi_{\mathrm{C}}^0 E^{-\zeta \ln E} \tag{15}$$

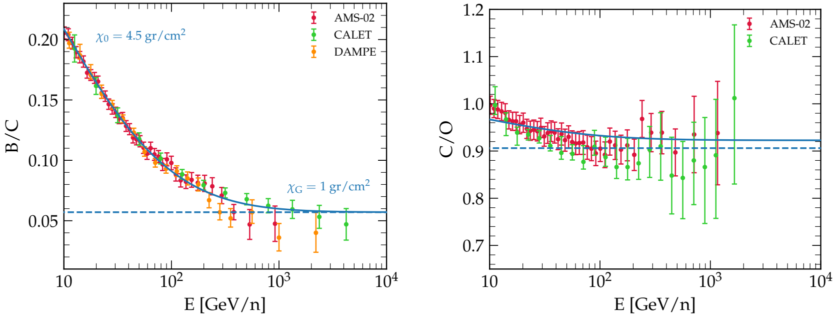
- Fixed model parameters

$$\mathcal{R} = 1/50 \text{ yr} \tag{16}$$
$$V_{\mathrm{G}} = \pi R_{\mathrm{G}}^2 2h \simeq 2 \times 10^{66} \text{ cm}^3 \tag{17}$$
$$E_0 = 10 \text{ GeV} \tag{18}$$
$$\sigma_{\mathrm{C}\to\mathrm{B}} = 60 \text{ mb} \tag{19}$$
$$\sigma_{\mathrm{O}\to\mathrm{B}} = 60 \text{ mb} \tag{20}$$
$$\sigma_{\mathrm{O}\to\mathrm{C}} = 60 \text{ mb} \tag{21}$$
$$\tag{22}$$

- Free model parameters

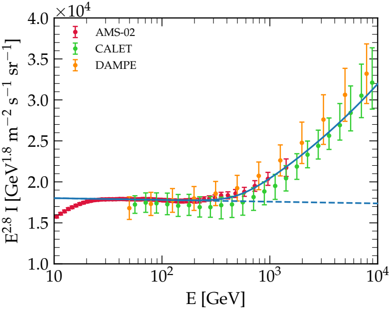
$$\chi_{\mathrm{G}} = [1, 10] \text{ gr/cm}^2 \tag{23}$$
$$\chi_{\mathrm{C}}^0 = [1, 10] \text{ gr/cm}^2 \tag{24}$$
$$\zeta = [0.01, 0.1] \tag{25}$$
$$\frac{N_{\mathrm{O}}}{N_{\mathrm{C}}} = [0.1, 1] \tag{26}$$
$$\tag{27}$$

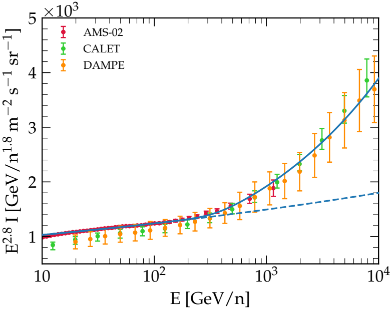
Boron-over-carbon



▷ From the B/C I fit $\tau_{\rm G} \simeq 1$ Myr (for $n_H = 0.5$) which is the energy loss timescale for 1 TeV positrons on CMB only.

Protons





- From the proton fit $\alpha = 2.81$, $\Delta\alpha = 0.20$, $E_b = 480$ GeV, $\xi = 2$%
- From the Helium fit $\alpha = 2.72$, $\Delta\alpha = 0.23$, $E_b = 330$ GeV, $\xi = 0.1$%

Beryllium

- Be10 is pure secondary and it decays with decay lifetime τ_d and is given by

$$N_{10} = \left(Q^{\mathrm{G}}_{10} + Q^{\mathrm{C}}_{10}\right) \frac{\tau_{\mathrm{G}} \tau_d}{\tau_{\mathrm{G}} + \tau_d} = c N_i \sigma_{i \to 10} \left(n_{\mathrm{G}} \tau_{\mathrm{G}} + n_{\mathrm{C}} \tau_{\mathrm{C}}\right) \frac{\tau_d}{\tau_{\mathrm{G}} + \tau_d} \tag{28}$$

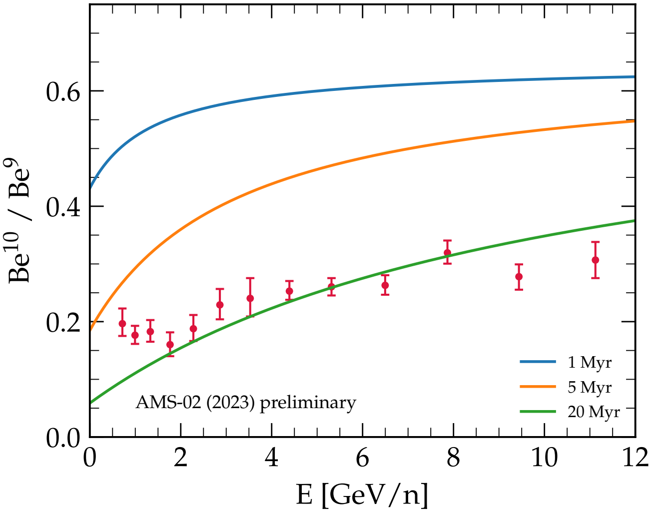
- following for the ratio

$$\frac{N_{10}}{N_9} = \frac{\sigma_{i \to 10}}{\sigma_{i \to 9}} \frac{\tau_{\mathrm{G}} + \tau_d}{\tau_d} \tag{29}$$

- or in case of more than one primary

$$\frac{N_{10}}{N_9} = \frac{\sigma_{i \to 10} + \frac{N_k}{N_i} \sigma_{k \to 10}}{\sigma_{i \to 9} + \frac{N_k}{N_i} \sigma_{k \to 9}} \frac{\tau_{\mathrm{G}} + \tau_d}{\tau_d} \tag{30}$$

Beryllium


0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
Be^{10} / Be^{9}
E [GeV/n]
AMS-02 (2023) preliminary
1 Myr
5 Myr
20 Myr

Antiprotons

- The antiproton emissivity is again given by the sum of two contributions

$$Q_{\bar{p}} = c n_{\mathrm{H}} \xi \int_{E_t(E)}^{\infty} dE' n_p(E') \frac{d\sigma(E,E')}{dE} \tag{31}$$

- while in the cocoon is

$$Q_{\bar{p},c}(E) = c n_{\mathrm{H,c}} \xi \int_{E_t(E)}^{\infty} dE' \rho_s q_i(E') \tau_{\mathrm{C}}(E') \frac{d\sigma(E,E')}{dE} \tag{32}$$

- therefore

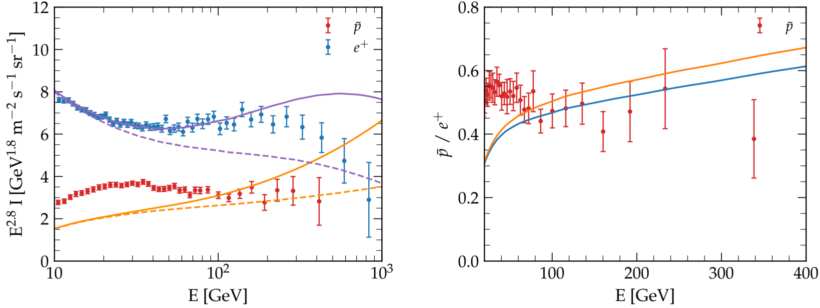
$$N_{\bar{p},\mathrm{G}} = Q_{\bar{p},\mathrm{G}} \tau_{\mathrm{G}} = \xi \int_{E_t(E)}^{\infty} dE' N_p(E') \frac{\chi_{\mathrm{G}}}{m_p} \frac{d\sigma(E,E')}{dE} \tag{33}$$

$$N_{\bar{p},\mathrm{c}} = Q_{\bar{p},\mathrm{c}} \tau_{\mathrm{G}} = \xi \int_{E_t(E)}^{\infty} dE' N_p(E') \frac{\chi_c(E')}{m_p} \frac{d\sigma(E,E')}{dE} \tag{34}$$

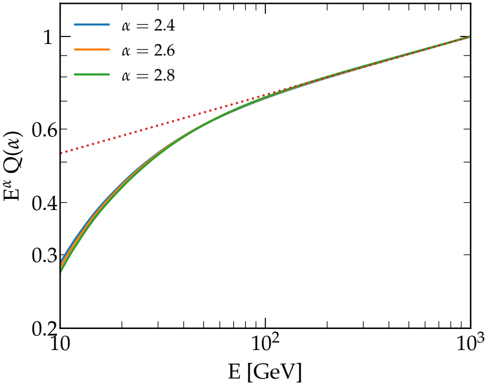
- For positrons

$$N_{e^+,\mathrm{G}} = Q_{e^+,\mathrm{G}} \frac{\tau_{\mathrm{G}} \tau_l}{\tau_{\mathrm{G}} + \tau_l} = \xi \frac{\tau_l(E)}{\tau_{\mathrm{G}} + \tau_l(E)} \int_{E_t(E)}^{\infty} dE' N_p(E') \frac{\chi_{\mathrm{G}}}{m_p} \frac{d\sigma(E,E')}{dE} \tag{35}$$

Anti-matter



Anti-matter



$$N_{\bar{p}} \propto E^{-\alpha_{\mathrm{p,He}}} E^{\beta_\sigma} E^{-\delta} \tag{36}$$

$$\frac{N_{\bar{p}}}{N_p} \propto \frac{E^{-\alpha_{\mathrm{p,H}}} E^{\beta_\sigma} E^{-\delta}}{E^{-\alpha_{\mathrm{p,L}}}} = E^{\Delta\alpha} E^{\beta_\sigma} E^{-\delta} \rightarrow 0 = 0.2 + 0.14 - \delta \tag{37}$$